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COMPUTATION OF THE SOUND ENERGY RADIATED FROM TURBULENT FLOWS

J. C. Rotta

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16. Abstract

The problem of predicting the distribution of radiated sound energy, when the distribution of mean velocity is known, is treated in two steps. Firstly, with the use of Lighthill's equation, the energy of sound is expressed as a function of kinetic fluctuation energy and a length scale. Secondly, the distributions of the required field quantities are determined with the aid of the transport equation for kinetic fluctuation energy. As example, computed results for the axisymmetric free jet and the free mixing layer are presented and discussed.

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COMPUTATION OF THE SOUND ENERGY RADIATED FROM TURBULENT FLOWS*

J. C. Rotta**

1. Introduction

The wave equation developed by J. M. Lighthill [1] forms the foundation for determining the acoustic energy radiated by turbulent fields. This equation states that the velocity fluctuations of a turbulent field in a medium at rest produce pressure and density fluctuations just like pulsating quadruples known from classical acoustics. However, the most important problem is to relate the intensities and distribution of the acoustic source terms to known parameters of the flow field. The strict solution of this problem is identical with the solution of the turbulence problem. /4***

Many authors have attempted to obtain information regarding the properties of turbulent fields using theoretical methods. However, only a few papers are known in which the investigations led to quantitative results. The paper given here deals with the problem of determining the acoustic source distribution from

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the distribution of the average velocity. This problem has considerable practical importance. A request was made to complement experimental investigations of the acoustic source distribution in turbulent jets (see the work of F. -R. Grosche [2]) by theoretical work.

The problem is solved in two steps. In the first step, the acoustic sources are determined from the solution of the Lighthill equation. An approximate method is used to determine the solution as a function of other field variables of turbulence, in particular, the kinetic energy of the velocity fluctuations and a characteristic length measure. In the second step, the variables are calculated using the transport equation for the kinetic fluctuation energy and other relationships, which are assumed to be known from the distribution of the average velocity.

2. Solutions of the Lighthill Equation /5

We are interested in the far field of the acoustic energy radiated by a region of turbulence having finite dimensions. Let

$$P\,(\underline{X}/X,\ \underline{y})\ d\underline{y} \tag{1}$$

be the acoustic power, which is radiated by a volume element $d\underline{y}$ ($= dy_1\ dy_2\ dy_3$) having the position $\underline{y}$ per unit of solid angle in the direction of the vector $\underline{X}$. The quantity $X = |\underline{X}|$ is the magnitude of the distance from the observation point to the turbulence region, and we assume that the dimensions of the turbulence region are small compared with X.

When the quantity P ($\underline{X}$/X, $\underline{y}$) is known, the total acoustic intensity at the position $\underline{X}$ is obtained by integration over the entire turbulence region.

Using the Lighthill wave equation, the solution of which is represented in terms of retarded potentials, we obtain the following approximation for P ($\underline{X}/X$, $\underline{y}$) (see H. S. Ribner [3, 4]):

$$P(\underline{X}/X, \underline{y}) = \frac{\rho_o X_i X_j X_k X_l}{16 \pi^2 a_o^5 X^4} \int \overline{\frac{\partial^2 (v_i v_j)}{\partial t^2} \frac{\partial^2 (v'_k v'_l)}{\partial t^2}} d\underline{r} \qquad (2)$$

This notation is based on the representation in the Cartesian coordinate system X_i ($i = 1, 2, 3$). The Einstein summation convention is adapted for indices which occur twice [$X = (X_i X_i)^{1/2}$]. v_i refers to the velocities (average value and fluctuation value) at the location $\underline{y}$, and v'_i are the velocities at $\underline{y} + \underline{r}$. The integral extends over the volume of the entire $\underline{r}$-space, $d\underline{r} = dr_1 dr_2 dr_3$. In addition, a_o is the speed of sound, and ρ_o is the density of the surrounding medium. Equation (2) contains the following assumptions:

1. In the medium surrounding the turbulence region, the small effects of heat conductivity and viscosity are ignored.

2. Outside of the turbulence retion, the motion is so small that the acoustic sources in the Lighthill equation can be set equal to zero ($\partial^2 T_{ij}/\partial t^2 \equiv 0$). /6

3. The average temperature in the turbulence field is not very different from the temperature of the surrounding medium.

4. The deviations from the adiabatic state conditions in the turbulence field are ignored.

5. The Reynolds number of the flow is large.

6. The Mach number is small ($Ma = [\overline{v^2}]^{1/2}/a$).

When the usual division of the velocities into an average value $\overline{u_i}$ and a fluctuation velocity u_i is made in Equation (2),

$$v_i = \overline{u_i} + u_i , \qquad (3)$$

we obtain two different components for the radiated acoustic energy. The first part is produced only by interactions of fluctuation velocities, and is called "eigensound" (self noise). The second part is produced by interactions of the turbulent fluctuation velocities with the average velocity. This part is called "shear sound" (shear noise).

2.1. Self noise

The summations of expressions of the type $X_i v_i/X$, $X_j v_j/X$, etc., contained in (2), state that only velocity components in the direction of the vector $\underline{X}$ have an effect. It is therefore appropriate to write the following for the self noise

$$P_e(\underline{X}/X, \underline{y}) = \frac{\rho_o}{16\pi^2 a_o^5} \int \overline{\frac{\partial^2 u_x^2}{\partial t^2} \frac{\partial^2 u_x'^2}{\partial t^2}} \, d\underline{r} \qquad (4)$$

where u_x and u_x' are the respective velocity fluctuations in /7
the direction $\underline{X}$. I. Proudman [5] gave an estimate of the self noise radiated by an isotropic turbulent field. Such a turbulent field radiates sound of equal intensity in all directions, so that P_e is independent of $\underline{X}/X$. From a dimensional analysis, we can specify the form

$$P_e(\underline{y}) = \frac{\rho_o}{a_o^5} \frac{\alpha}{4\pi} \frac{(\overline{u^2})^4}{L_1} \tag{5}$$

where $\overline{u^2}$ is the mean square value of the velocity fluctuations which is equal in all directions. L_1 is the integral length measure (integral over the two-point correlation function) (see [6]).

$$L_1 = \int_0^\infty f(r)\, dr\,. \tag{6}$$

The calculation of the quantity α requires a number of assumptions and long algebraic calculations, which do not have to be repated here. First, the cumulants of fourth order are ignored, so that the correlations of four velocity components can be expressed as products of correlations of two velocity components each. The time derivatives are determined using the Navier-Stokes equations of motion. Again, the fourth order cumulants are ignored in order to eliminate the fluctuations of pressure in the equations. According to Proudman, we find the following results for a steady turbulence field

$$\alpha = \frac{8}{5}\int_0^\infty (fG)^2 x^4\, dx - \frac{8}{15}\int_0^\infty \frac{df}{dx}\left\{G\int_x^\infty fG\, dx' - f\frac{d}{dx}\left[\frac{d(fG)}{dx} + 4\frac{fG}{x}\right]\right\} x^4\, dx \tag{7}$$

where /8

$$G = \frac{d}{dx}\left(\frac{d^2 f}{dx^2} + 4\,\frac{df/fx}{x}\right) \tag{8}$$

and $x = r/L_1$. The numerical evaluation of this formula resulted in $\alpha = 37.5$, using the function f tabulated by I. Proudman [7] for very large Reynolds numbers.

2.2. Shear noise

For the case of shear noise, we assume a simple, steady shear flow in which the average velocity $\bar{u}$ has the direction of the x_1 axis and is a function of x_2. If (3) is substituted in (2), then only those components in which i, j, k or l take on the value 1 will make a contribution to the shear hoise. H. S. Ribner [4] showed that most of these terms drop out, because of continuity or symmetry.

We obtain the following from (2) for the radiated acoustic power:

$$P_s(\underline{X}/X, y) = \frac{\rho_o}{16\pi^2 a_o^5}\left[4\left(\frac{X_1}{X}\right)^4 \int \overline{u}\,\overline{u'}\; \overline{\frac{\partial^2 u_1}{\partial t^2}\,\frac{\partial^2 u'_1}{\partial t^2}}\, d\underline{r} + 4\,\frac{X_1^2 X_3^2}{X^4}\int \overline{u}\,\overline{u'}\;\overline{\frac{\partial^2 u_3}{\partial t^2}\,\frac{\partial^2 u'_3}{\partial t^2}}\, d\underline{r}\right] \tag{9}$$

Again, the flow velocities must be taken at the point $\underline{y}$ and quantities having a bar must be evaluated at the point $\underline{y} + \underline{r}$.

If we also assume the isotropic tensor form for the correlation function $\overline{(\delta^2 u_i/\delta t^2)(\delta^2 u'_j/\delta t^2)}$, then we have*

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*Strictly speaking, it is sufficient for the correlation function to have an axisymmetric tensor form with the axis x_2.

$$\int \overline{u u'} \, \overline{\frac{\partial^2 u_3}{\partial t^2} \frac{\partial^2 u'_3}{\partial t^2}} \, d\underline{r} = \int \overline{u u'} \, \overline{\frac{\partial^2 u_1}{\partial t^2} \frac{\partial^2 u'_1}{\partial t^2}} \, d\underline{r} \quad . \tag{10}$$ /9

In this way this expression simplifies (9) to

$$P_s(\underline{X}/X, \underline{y}) = \frac{\rho_o}{4\pi^2 a_o^5} \frac{X_1^4 + X_1^2 X_3^2}{X^4} \int \overline{u u'} \, \overline{\frac{\partial^2 u_1}{\partial t^2} \frac{\partial^2 u'_1}{\partial t^2}} \, d\underline{r} \quad . \tag{11}$$

According to the present state of knowledge on the structure of turbulence, the assumption of an isotropic form of the two-point correlation seems to be almost the only way which will allow a quantitative evaluation of the integral. The direction dependence of P_s is then only expressed in the factor

$$k(\underline{X}/X) = \frac{X_1^4 + X_1^2 X_3^2}{X^4} \tag{12}$$

$(0 \leqq k \leqq 1)$. In the case of axisymmetric flows, the direction of the vector $\underline{X}$ is described by angular coordinates. It is then possible to average over the azimuth angle, and we obtain the directional dependence obtained by Ribner [4].

$$k(\underline{X}/X) = \frac{\cos^4 \Theta + \cos^2 \Theta}{2} \quad . \tag{13}$$

if Θ is the angle between the vector $\underline{X}$ and the jet axis.

The integral expression can be represented in the following form based on dimensional analysis

$$\int \overline{u u'} \, \overline{\frac{\partial^2 u_1}{\partial t^2} \frac{\partial^2 u'_1}{\partial t^2}} \, d\underline{r} = \overline{u}^2 \frac{(\overline{u^2})^3}{L_1} \beta \tag{14}$$

where the coefficient β depends on the distribution of the average velocity. In contrast to α in (5), it is a function of /10 position for a given flow. In this way, Equation (11) is reduced to the form

$$P_s (\underline{X}/X, \underline{y}) = \frac{\rho_o}{a_o^5} k (\underline{X}/X) \frac{\beta}{4\pi^2} \overline{u}^2 \frac{(\overline{u^2})^3}{L_1} . \tag{15}$$

The calculation of β uses the same assumptions and restrictions as were used for α in (5). We can again use the relationship given by Proudman. After a few calculations (see Appendix A), we obtain

$$\beta = 2\pi \int_0^\infty \left\{ \phi_1 (y, r) \left[\frac{x}{4} F' (x) + F \right] + \phi_2 (y, r) \frac{x}{4} F' (x) \right\} x^2 dx \tag{16}$$

with

$$F' (x) = - G \int_x^\infty f G d\xi + f \frac{d}{dx} \left[\frac{d}{dx} (f G) + 4 \frac{f G}{x} \right] , \tag{17}$$

$$F = - \int_x^\infty F' d\xi , \tag{18}$$

$$\phi_1 (y, r) = \int_0^1 \left[\frac{\overline{u} (y + L_1 x z) + \overline{u} (y - L_1 x z)}{\overline{u} (y)} - 2 \right] dz, \tag{19}$$

$$\phi_2 (y, r) = \int_0^1 \left[\frac{\overline{u} (y + L_1 x z) + \overline{u} (y - L_1 x z)}{\overline{u} (y)} - 2 \right] z^2 dz . \tag{20}$$

G and f have the same meaning as in Equations (7) and (8). The numerical evaluation is again based on the correlation function given by Proudman [7] for large Reynolds numbers.

As can be seen from Equation (19) and (20), the distribution of the average velocity influences the value of β, in

particular because of the local curvature. For negative /11
$\delta^2\bar{u}/\delta y^2$, that is, along the free jet axis, we find positive β. For positive $\delta^2\bar{u}/\delta y^2$, i.e., at the outer edge of the jet, we find in part very large negative β values, which do not have any influence because of the low fluctuation energy and low average velocities at this point.

3. Kinetic Energy of the Fluctuation Velocities

In the previous section, we treated the dependence of the radiated acoustic power on the kinetic energy of the velocity fluctuations and an integral length measure. We will now investigate the question of how the distributions of the kinetic fluctuation energy and the length measure are related with the distribution of the average velocity. Recently, many attempts have been made to calculate the distributions of the average velocities using the transport equation for the kinetic fluctuation energy or other transport equations. These methods provide as a side result the field quantities which are required for estimating the acoustic sources. However, there is also the possibility of calculating the distribution of the kinetic fluctuation energy, if the distribution of the average velocity is known. The principle is to calculate the normal component of the average velocity using the continuity equation and to also calculate the shear stress distribution using the equation of motion. The quantities obtained in this way are then introduced in the energy equation. The transport equation for the kinetic fluctuation energy $\overline{q^2}/2 = (\overline{u^2} + \overline{v^2} + \overline{w^2})/2$ is as follows for large Reynolds numbers using the usual boundary layer simplifications (see [6])

$$\bar{u}\,\frac{\partial(\overline{q^2}/2)}{\partial x} + \bar{v}\,\frac{\partial(\overline{q^2}/2)}{\partial y} + \overline{u\,v}\,\frac{\partial \bar{u}}{\partial y} + \epsilon + \frac{1}{y^j}\,\frac{\partial}{\partial y}\left[y^j\overline{\left(\frac{q^2}{2} + p\right)v}\right] = 0 \qquad (21)$$

where

$j = 1$ for axisymmetric flows

and /12

$j = 0$ for plane flows

and we have set $\rho = 1$.

If we desire to calculate the distribution of $\overline{q^2}/2$ for known average velocity fields and known Reynolds stress $-\overline{u'v'}$, then it is necessary to introduce trial solutions for the turbulent energy dissipation ε and for the turbulent energy diffusion $\overline{(q^2/2 + p)v}$. We can use the following relationships for these two quantities

$$\epsilon = c\left[\overline{q^2}/2\right]^{3/2}/L, \qquad (22)$$

$$\overline{(q^2/2 + p)v} = -k_q\sqrt{\overline{q^2}/2}\,L\,\partial(\overline{q^2}/2)/\partial y \qquad (23)$$

where L, the length measure of turbulence, is a field variable and k_q and c are dimensionless coefficients. In order to be able to solve Equation (21), c and k_q must be known, as well as the distribution of L. The system of equations is determined by adding another relationship for $\overline{u\,v}$, which however cannot contain any other flow parameters as variables. We will use the following exchange trial solution of L. Prandtl [8] for this relationship

$$\overline{uv} = -k\sqrt{\overline{q^2}/2}\, L\, \partial\bar{u}/\partial y \quad (24)$$

where k is another dimensionless coefficient. If we assume that k is known, then the quantity $\sqrt{\overline{q^2}/2}\, L$ is calculated from the velocity field by inverting Equation (24). This means that L can be eliminated from (22) and (23), so that (21) is transformed into a partial differential equation for $\overline{q^2/2}$ after (22) and (23) are introduced. We obtain the following relationship from the requirement that in the vicinity of fixed walls, the relationship must be compatible with the universal velocity law. /13

$$k^3 = c\,. \quad (25)$$

We reported on this method of calculating the kinetic fluctuation energy and the length measure elsewhere [9]. As example, we treated the fully developed pipe flow, the plane asymptotic wake flow and the plane free jet in a medium at rest. Here we will give the results for the round free jet in a medium at rest and the results for the free jet boundary. Details of the calculation are described in Appendix B.

4. Results and Discussion

Figure 1 shows the distributions of the kinetic fluctuation energy and of the length measure for round free jets in the dimensionless form as a function of y/b for various values of the coefficient k_q of energy diffusion. u_1 is the average velocity along the axis, and b is the radius at which $\bar{u} = u_1/2$. In the inner region, the distributions only slightly depend on k_q. As k_q increases, there is a greater distribution of the energy. The calculated values of $\overline{q^2}/(2u_1^2)$ are about 20% lower than those determined in measurements of I Wygnanski and H. Fiedler [10].

Figure 2 shows the distribution of $\overline{q^2}/(2u_o^2)$ and L/x for the free jet boundary as a function of $\xi = \sigma y/x$, where σ = 12; u_o is the velocity at the nozzle exit. The distribution of the average velocity is also shown. The variation of k_q has the same effect as for a free jet. The values of $\overline{q^2}/(2u_o^2)$ agree well with the experimental results of I. Wygnansky and H. Fiedler [11].

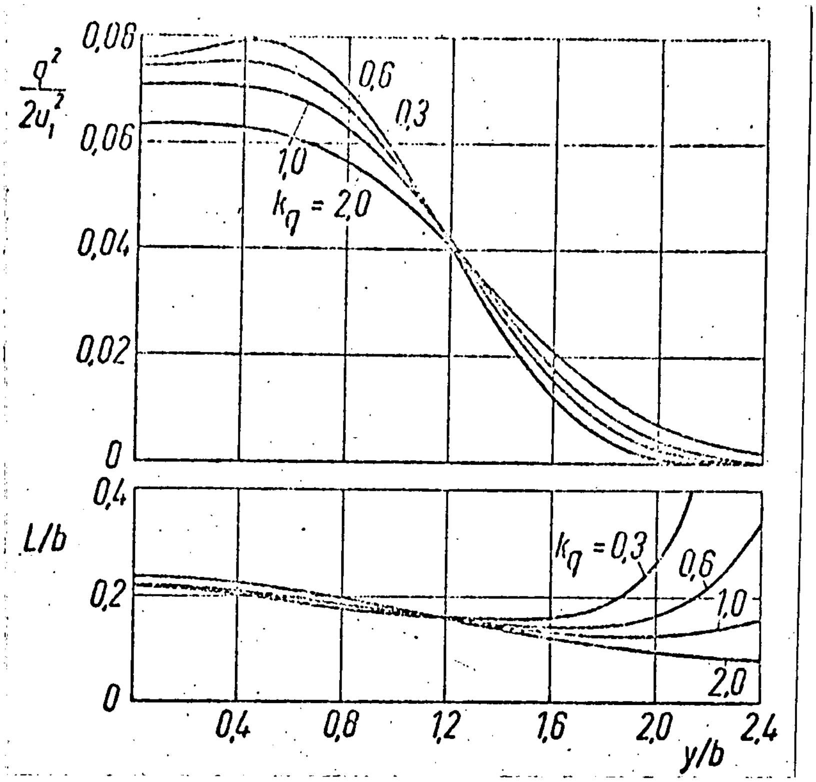

Figure 1. Round free jet in surroundings at rest b/x = 0.88; c = 0.165

a — distribution of the fluctuation energy; b — distribution of the length measure

The distributions of the length measure are approximately constant over a wide range, but increase quite suddenly near the edges. This phenomenon becomes more pronounced, the smaller k_q. /14

As already determined in an earlier work [9], the agreement between computation results and experiment is good in many cases.

In other cases it is not as good. It is possible that better results could be obtained if the exchange law (24) is replaced by the transport equation for the Reynolds shear stress. This question remains to be investigated.

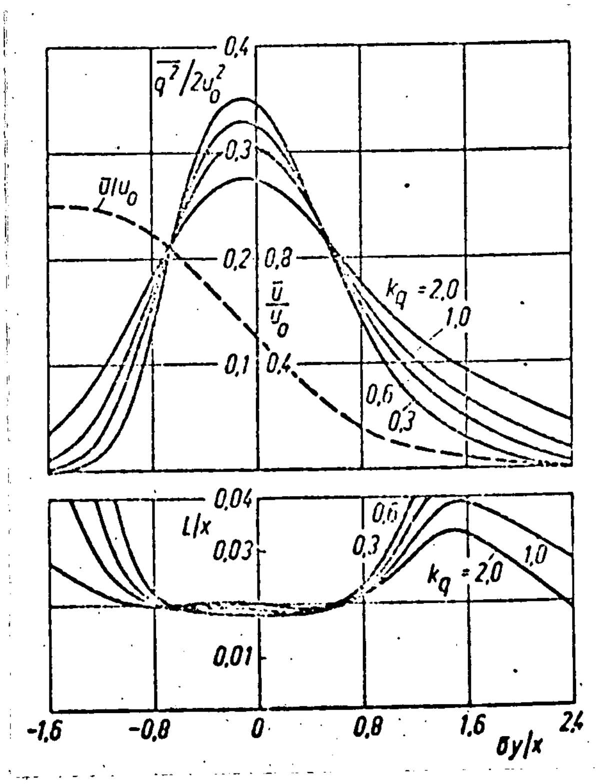

Figure 2. Free jet boundary ($u_\infty = 0$), $\sigma = 12$, $c = 0.165$

When calculating the acoustic source distributions, it is necessary for the length measure L in the energy equation, i.e., in Equations (22), (23), and (24), to correspond to the integral for the transverse correlation function as given in the definition. Therefore, it will be half as large as L_1, the integral over the length correlation function. Therefore, we set

$$L_1 = 2L \tag{26}$$

The mean square average of the velocity fluctuations which occur in (5) and (15) is expressed in terms of the kinetic energy of the fluctuations:

$$\overline{u^2} = \frac{2}{3}\,\overline{q^2}/2 \quad . \tag{27}$$

The quantity P (X/X, y) is transformed into a suitable form for a dimensionless representation of the radiated acoustic energy. For the free jet, we write

$$P(\underline{X}/X, \underline{y}) = \frac{\rho_o u_1^8}{a_o^5 b} k(\underline{X}/X) I(y/b) \tag{28}$$

for the free jet boundary

$$P(\underline{X}/X, \underline{y}) = \frac{\rho_o u_o^8}{a_o^5 x} k(\underline{X}/X) I(y/x) , \tag{29}$$

where the directional coefficient k ($\underline{X}/X$) for the self sound is k ≡ 1.

Figure 3 shows the dimensionless distributions of the self-sound sources I_c (y/b) and the shear sound sources I_s (y/b). The sound sources concentrate along the jet axis, as was to be expected. The shear sound has a much smaller intensity according to these calculations than does the self sound. This is in contradiction to the estimates of H. S. Ribner [4], according to which the peak value of the shear sound is approximately equal to the peak value of the self sound. Because of the very different assumptions, it is not possible to make a direct comparison. /15

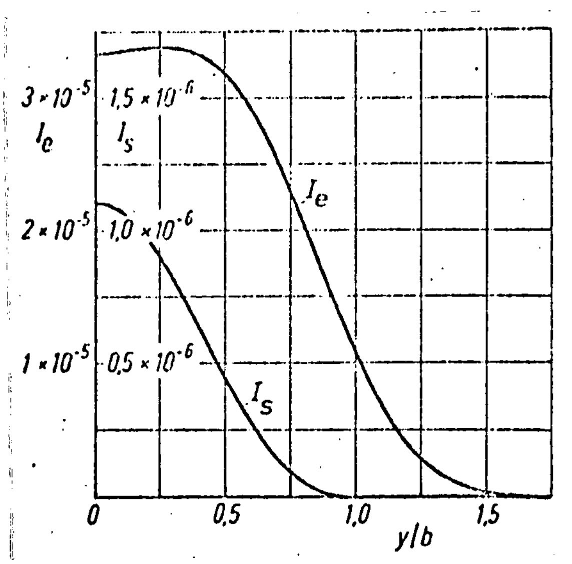

Figure 3. Distribution of the acoustic sources in the round free jet, k_q = 1, c = 0.165

The calculated distributions of the sound sources are given in Figure 4 for the free jet boundary. The self noise is produced in a relatively narrow region in which the gradient of the averaged velocity is the steepest. The shear sound sources I_s have positive as well as negative values because of the course of β. Compared with the self sound sources, they are considerably smaller than in the case of the free jet. This behavior must be attributed to the fact that the sign change of β (approximately at $\delta^2\bar{u}/\delta y^2 = 0$) occurs in the region of maximum fluctuation intensity. The following remarks can be made regarding these calculations:

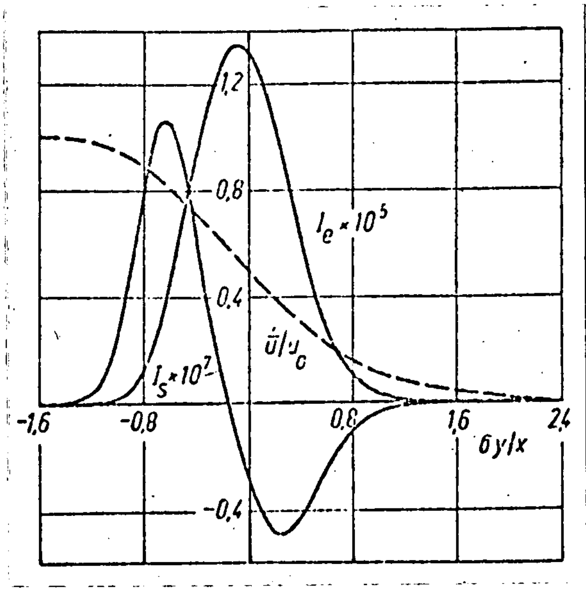

Figure 4. Distribution of the acoustic sources in the free jet boundary, $\sigma = 12$, $k_q = 1$, $c = 0.165$

1. The influences of convection of the vortex structure and of refraction by the flow field were not considered in the results discussed above. This means that extremely small flow velocities have been assumed. As Lighthill, Ribner, and others have shown, these effects, which change the intensity and direction characterics of the acoustic radiations significantly, can be approximately taken into account by means of correction factors. We will not discuss the details of this.

2. The division into "self sound" and "shear sound" made in Equation (4) and (9) is somewhat arbitrary. The correlation functions in (4) and (9) depend implicitly on the average velocities and their derivatives because of

equations of motion. If a calculation is made according to I. S. F. Jones [12], in which the time derivatives under the integral of (2) are substituted into the equations of motion before decomposition of the velocities into a mean value and a fluctuation value, one obtains completely different results for the shear sound than what was obtained above, which was based on Ribner's work. /16

3. Reservations about the preceding calculation can be made based on the fact that the average velocity was ignored and isotropic distributions were assumed in the determination of the time derivatives. One point of view which supports these assumptions is the fact that 75% of the calculated self sound emerges from an area in which the correlation function f is larger than 0.76. As is well known, and assuming large Reynolds numbers, there is a universal and locally isotropic structure which is only influenced slightly by the average velocity. Conditions are not as favorable for the shear sound, but the contributions for this case are relatively small. These arguments do not mean that no substantial contributions could be made, because of the effect of averaged shear velocities and the consequent change in the turbulence structure. The simplifications made are accepted because at the present time they represent the only way of progressing towards quantitative results.

4. According to experimental investigations, there are other ordered types of motion in addition to the unordered turbulence motion. Such vortex shapes are quite clear in flow photographs within a mixing region having large density differences. G. Brown and A. Roshko [13] have

shown this. According to the work of A. Michalke [14], H. V. Fuchs [15], as well as K. A. Bishop et al. [16], these motions contribute significantly to sound production, especially at high flow velocities. These contributions are not described by the methods mentioned here. Unfortunately, we were not able to use the theoretical developments of Michalke. We hope that perhaps later on it will be possible to construct a useful wave model for the acoustic production, using turbulence variables established by means of the transport equations for kinetic energy and Reynolds stresses. /17

In light of these arguments, our investigations can only be looked upon as a first attempt of determining the acoustic energy radiated by turbulent fields, once the corresponding distribution of the average velocity is known. We hope that we have generated some interest in this problem. Further work, using refined assumptions, is necessary.

5. Summary

We calculated the acoustic source distributions in turbulent flows for the case where the average velocity distribution was known. The basic idea is to determine characteristic variables using the transport equation for the kinetic energy of the fluctuations and other relationships. These characteristic variables describe the turbulence structure. Starting with the Lighthill equation, we expressed the radiated acoustic energy as a function of the kinetic fluctuation energy, an integral length measure and the distribution of the average velocity. These calculations in which the relationships for isotropic turbulence fields were used continue the investigations of I. Proudman. As an illustration, we applied the relationships to a round free jet in a medium at rest and to the free jet boundary. The calculated distributions of kinetic fluctuation energy of the length measure and the acoustic sources are discussed and a critical evaluation of the physical assumptions is made.

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APPENDIX A: SHEAR NOISE /20

When the quantity β is determined in (15), we will for the most part use the relationships of I. Proudman [5]. The equation numbers of this paper [5] are given by means of double brackets. We start with the isotropic, tensor form

$$\overline{\frac{\partial^2 u_i}{\partial t^2}\frac{\partial^2 u_j}{\partial t^2}} = \overline{\left(\frac{\partial^2 u}{\partial t^2}\right)^2}\left[-\frac{1}{2r}\psi' r_i r_j + \left(\frac{1}{2} r\psi' + \psi\right)\delta_{ij}\right] . \qquad \begin{matrix}(A.1)\\((5.2))\end{matrix}$$

ψ is a dimensionless function of the distance r_j. Primed values refer to derivatives with respect to r. Then we have

$$\overline{\left(\frac{\partial^2 u}{\partial t^2}\right)^2}\psi' = -\overline{u^2}\,\overline{\left(\frac{\partial u}{\partial t}\right)^2}\left[\psi\frac{d}{dr}\left(f'' + 4\frac{f'}{r}\right) + f\frac{d}{dr}\left(\psi'' + 4\frac{\psi'}{r}\right)\right] \qquad \begin{matrix}(A.2)\\((5.17))\end{matrix}$$

and also, according to ((5.9))

$$\overline{\left(\frac{\partial u}{\partial t}\right)^2}\,\varphi' = -\left(\overline{u^2}\right)^2 f\frac{d}{dr}\left(f'' + 4\frac{f'}{r}\right) = -\frac{\left(\overline{u^2}\right)^2}{L_1^3} fG\ , \tag{A.3}$$

where G is the expression defined by Equation (8) $(x = r/L_1)$.

It follows

$$\overline{\left(\frac{\partial u}{\partial t}\right)^2}\,\varphi = -\overline{\left(\frac{\partial u}{\partial t}\right)^2}\int_r^\infty \varphi'\,dr = \frac{\left(\overline{u^2}\right)^2}{L_1^2}\int_x^\infty fG\,d\xi\ . \tag{A.4}$$

If these relationships are substituted in (A.2), we obtain

$$\overline{\left(\frac{\partial^2 u}{\partial t^2}\right)^2}\,\psi' = \frac{\left(\overline{u^2}\right)^3}{L_1^5} F'(x) \tag{A.5}$$

with /21

$$F'(x) = \frac{dF}{dx} = -G\int_x^\infty fG\,d\xi + f\frac{d}{dx}\left[\frac{d}{dx}(fG) + 4\frac{fG}{x}\right]\ . \tag{A.6}$$

Also, we find

$$\overline{\left(\frac{\partial^2 u}{\partial t^2}\right)^2}\,\psi = -\overline{\left(\frac{\partial^2 u}{\partial t^2}\right)^2}\int_r^\infty \psi'\,dr = -\frac{\left(\overline{u^2}\right)^3}{L_1^4}\int_x^\infty F'\,d\xi = \frac{\left(\overline{u^2}\right)^3}{L_1^4} F(x)\ . \tag{A.7}$$

After substituting (A.5) and (A.7) in (A.1), we then find

$$\overline{\frac{\partial^2 u_i}{\partial t^2}\,\frac{\partial^2 u'_j}{\partial t^2}} = \frac{\left(\overline{u^2}\right)^3}{L_1^4}\left\{-F'(x)\frac{r_i r_j}{2rL_1} + \left[\frac{1}{2}F'(x)\,x + F(x)\right]\delta_{ij}\right\} \tag{A.8}$$

and for the special case

$$\overline{\frac{\partial^2 u_1}{\partial t^2}\,\frac{\partial^2 u_1'}{\partial t^2}} = \frac{(\overline{u^2})^3}{L_1^4}\left\{F'(x)\left[\frac{x}{2} - \frac{r_1^2}{2xL_1^2}\right] + F(x)\right\} . \tag{A.9}$$

In order to carry out the integration of (14), we introduce spherical coordinates:

$$\left.\begin{aligned} r_1 &= r \sin\vartheta \sin\varphi , \\ r_2 &= r \cos\vartheta , \\ d\underline{r} &= r^2 \sin\vartheta \, d\varphi \, d\vartheta \, dr . \end{aligned}\right\} \tag{A.10}$$

Here ϕ varies over the range between 0 and 2π and ϑ varies over the range from 0 to π. We will make the substitution $z = \cos$ so that

$$\left.\begin{aligned} r_1^2 &= r^2(1-z^2)\sin^2\varphi , \\ r_2 &= r \cdot z , \\ d\underline{r} &= -r^2 \, d\varphi \, dz \, dr \end{aligned}\right\} \tag{A.11}$$

and z runs from 1 to - 1. We can first carry out the integration over ϕ. Since because of the continuity equation

$$\int \overline{\frac{\partial^2 u_1}{\partial t^2}\,\frac{\partial^2 u_1'}{\partial t^2}} \, d\underline{r} \equiv 0 , \tag{A.12}$$

we introduce the following auxiliary functions for the integration over z:

$$\left.\begin{aligned}\phi_1(y, r) &= \int_0^1 \left[\frac{\bar{u}(y+\frac{r}{2}) + \bar{u}(y-\frac{r}{2})}{\bar{u}(y)} - 2\right] dz ,\\ \phi_2(y, r) &= \int_0^1 \left[\frac{\bar{u}(y+\frac{r}{2}) + \bar{u}(y-\frac{r}{2})}{\bar{u}(y)} - 2\right] z^2 dz\end{aligned}\right\} \quad (A.13)$$

and we then obtain the expression given in Equation (16) for β.

APPENDIX B: KINETIC ENERGY OF THE FLUCTUATIONS

1. Round free jet

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The well developed round free jet propagates linearly with distance from the exhaust point, and the distributions of the statistical quantities are similar at all intersection points perpendicular to the jet axis. The reference velocity u_1 is the averaged velocity along the axis and the reference length is the radial distance from the axis, in which the average velocity $\bar{u}$ equals $u_1/2$. This means that

$$b = \alpha x . \quad (B.1)$$

According to experimental results, we have set $\alpha = 0.088$ for the constant. Using $\eta = y/b$, we describe the average velocity by $\bar{u} = u_1 f(\eta)$. The same relationship for $f(\eta)$ was used as for the plane free jet (see [9]):

$$f(\eta) = \exp\{-0,6749\,\eta^2(1+0,0269\,\eta^4)\} \quad (B.2)$$

We assumed the following trial solutions for the fluctuation energy $\overline{q^2}/2$ and the length measure L

$$\overline{q^2}/2 = u_1^2 \, \psi(\eta) ; \quad L = b \lambda(\eta) \tag{B.3}$$

where ψ (η) and λ (η) are dimensionless functions of η. Using the similarity trial solutions, the partial differential equations for the average velocity and the fluctuation energy become ordinary differential equations, which can be solved using a Runge-Kutta method (see [9]).

2. Free jet boundary

The width of the turbulent mixing zone of a plane free jet boundary also grows linearly with distance from the exhaust point. The reference length is the distance x from the exhaust point, and the reference velocity is the exhaust velocity u_o. The dis- /24 tribution of the average velocity $\bar{u}$ is assumed in the form

$$\bar{u} = u_o \, F(\xi) \tag{B.4}$$

with $\xi = \sigma y/x$. σ is a constant. We assumed the following approximation for F (ξ)

$$F(\xi) = 1 - \frac{1}{2} \left[1 + \operatorname{erf}(0,9\ \xi)\right] \frac{1 + \operatorname{erf}\{2(\xi + 1)\}}{1 + \operatorname{erf}(2)} + 0,03\, \xi^4 \exp(-\xi^2) \left[1 + \operatorname{erf}\{0,8(\xi - 0,8)\}\right] \tag{B.5}$$

This relationship agrees well with the measurements of H. W. Liepmann*, if we set $\sigma = 12$. In addition, we introduced the following similarity trial solutions:

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*See H. W. Liepmann, John Laufer Investigations of Free Turbulent Mixing, NACA TN No. 1257 (1947).

$$\left.\begin{aligned} \overline{v} &= u_o\, G(\xi), \\ -\overline{uv} &= u_o^2\, \varphi(\xi), \\ \overline{q^2}/2 &= u_o^2\, \psi(\xi), \\ L &= x\, \lambda(\xi). \end{aligned}\right\} \qquad (B.6)$$

With these trial solutions, the partial differental equations for the flow are again reduced to ordinary differental equations, which can be numerically integrated using a Runge-Kutta method.

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